EE 508
Lecture 28

Integrator Design

Current-Mode Integrators

s-domain to z-domain mappings

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$A_V(j\omega)=\frac{\pm 1}{R(\omega)+jX(\omega)}$$

where R(ω) and X(ω) are real and represent the real and imaginary parts of the denominator respectively

$$\text{Phase} = -\tan^{-1}\left(\frac{X(\omega)}{R(\omega)}\right)$$

Ideally R(ω)=0

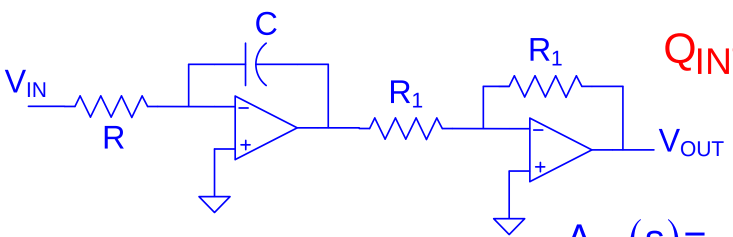
Definition: The Integrator Q factor is the ratio of the imaginary part of the denominator to the real part of the denominator

$$Q_{INT} = \left(\frac{X(\omega)}{R(\omega)}\right)$$

Typically most interested in Q_{INT} at the nominal unity gain frequency of the integrator

Review from last time

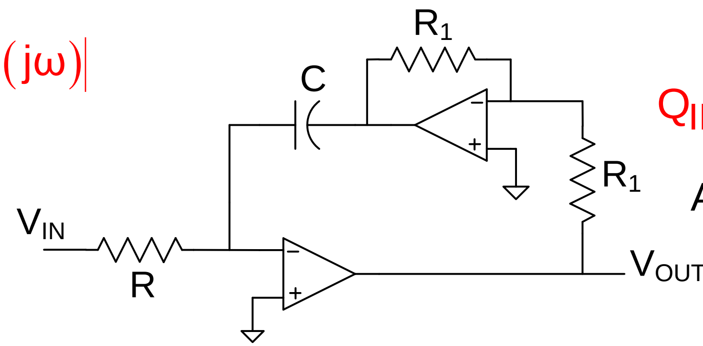
What are the integrator Q factors for other integrators that have been considered?



$Q_{INT} \simeq -\frac{1}{3}\left|A(j\omega)\right|$

$A_V(s) = \frac{1}{RCs}$

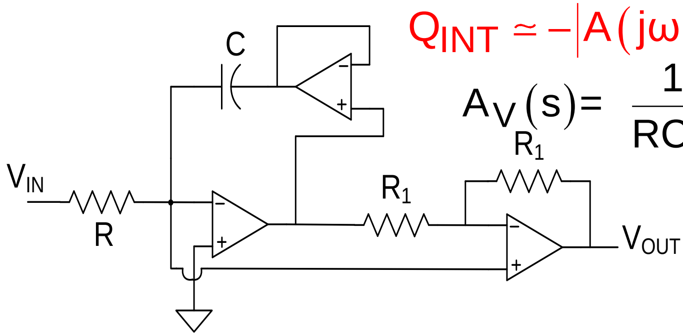
Miller Noninverting



$Q_{INT} \simeq \left|A(j\omega)\right|$

$A_V(s) = \frac{1}{RCs}$

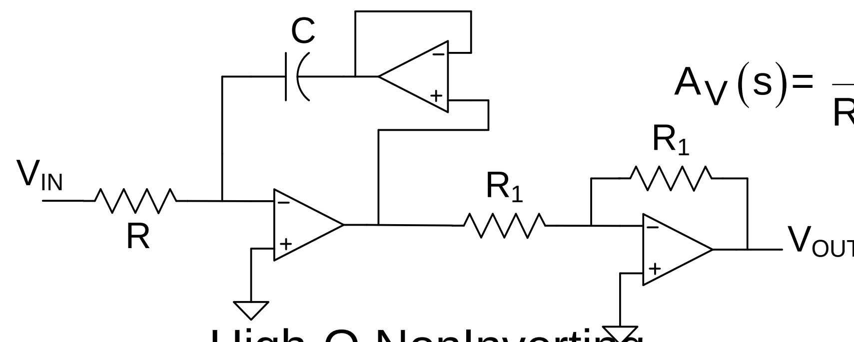
Phase Lead Integrator



$Q_{INT} \simeq -\left|A(j\omega)\right|^3$

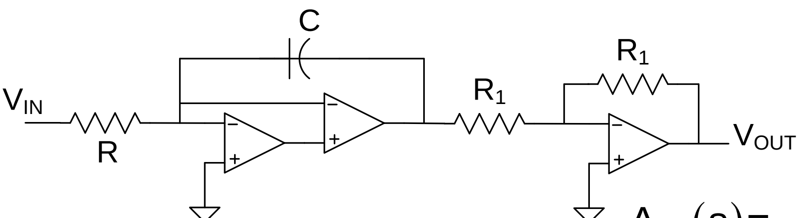
$A_V(s) = \frac{1}{RCs}$

Modified High-Q Noninverting



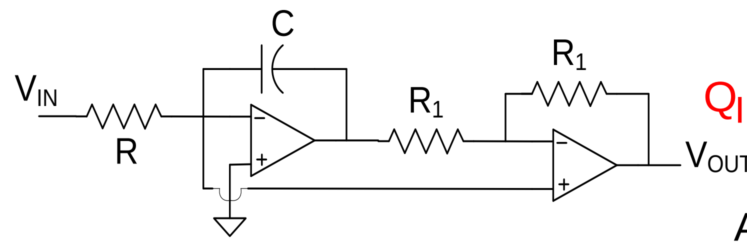
$A_V(s) = \frac{1}{RCs}$

High-Q NonInverting



$A_V(s) = \frac{1}{RCs}$

Zero Sensitivity Noninverting



$Q_{INT} \simeq -\left|A(j\omega)\right|$

$A_V(s) = \frac{1}{RCs}$

Modified Miller Noninverting

Review from last time

Improving Integrator Performance:

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

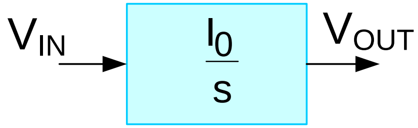
- These methods all provide some improvements in integrator performance

- But both magnitude and phase of an integrator are important so focusing only on integrator Q factor only may only improve performance to a certain level

- In higher-order integrator-based filters, the linearity in $1/\omega$ of the integrator gain is also important. The integrator magnitude and Q factor at ω_0 ignore the frequency nonlinearity that may occur in the $1/\omega$ dependence

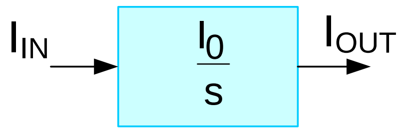
- There is little in the literature on improving the performance of integrated integrators within a basic class. At high frequencies where the active device performance degrades, particularly in finer-feature processes, there may be some benefits that can be derived from architectural modifications along the line of those discussed in this lecture

Integrator Types



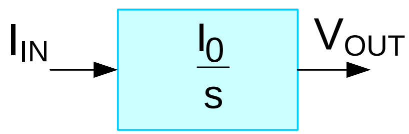
Voltage Mode

$$V_{OUT} = \frac{I_o}{s} V_{IN}$$



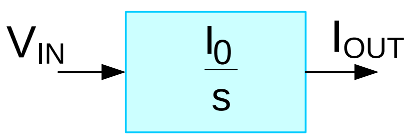
Current Mode

$$I_{OUT} = \frac{I_o}{s} I_{IN}$$



Transresistance Mode

$$V_{OUT} = \frac{I_o}{s} I_{IN}$$

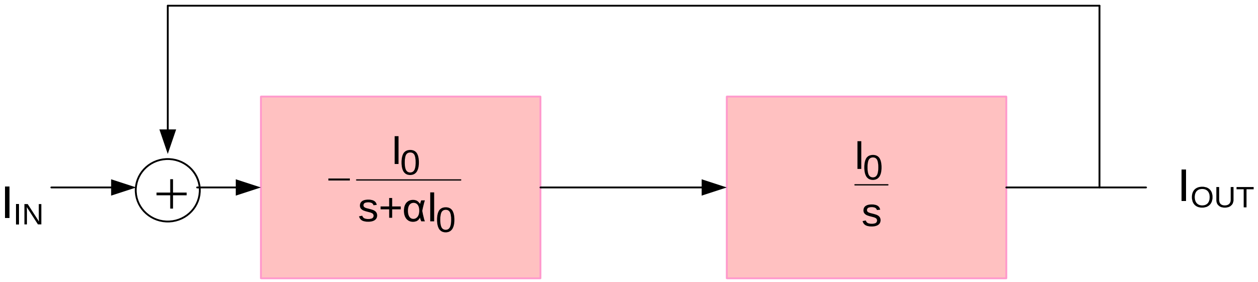


Transconductance Mode

$$I_{OUT} = \frac{I_o}{s} V_{IN}$$

Selected Current Mode, Transresistance Mode, and Transconductance Mode Integrators

Current-Mode Filters

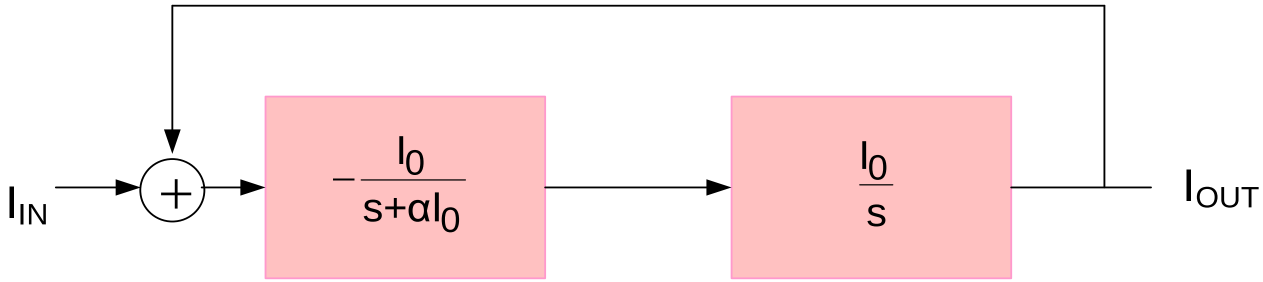


$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Basic Concepts of Benefits of Current-Mode Filters:

- Large voltage swings difficult to maintain in integrated processes because of linearity concerns
- Large voltage swings slow a circuit down because of time required to charge capacitors
- Voltage swings can be very small when currents change
- Current swings are not inherently limited in integrated circuits (only voltage swings)
- With low voltage swings, current-mode circuits should dissipate little power

Current-Mode Filters



$$T(s)=\frac{I_{OUT}}{I_{IN}}=\frac{-I_0^2}{s^2+\alpha I_0 s+I_0^2}$$

Concept of Current-Mode Filters is Somewhat Recent:

Key paper that generated interest in current-mode filters (ISCAS 1989):

Switched currents-a new technique for analog sampled-data signal processing

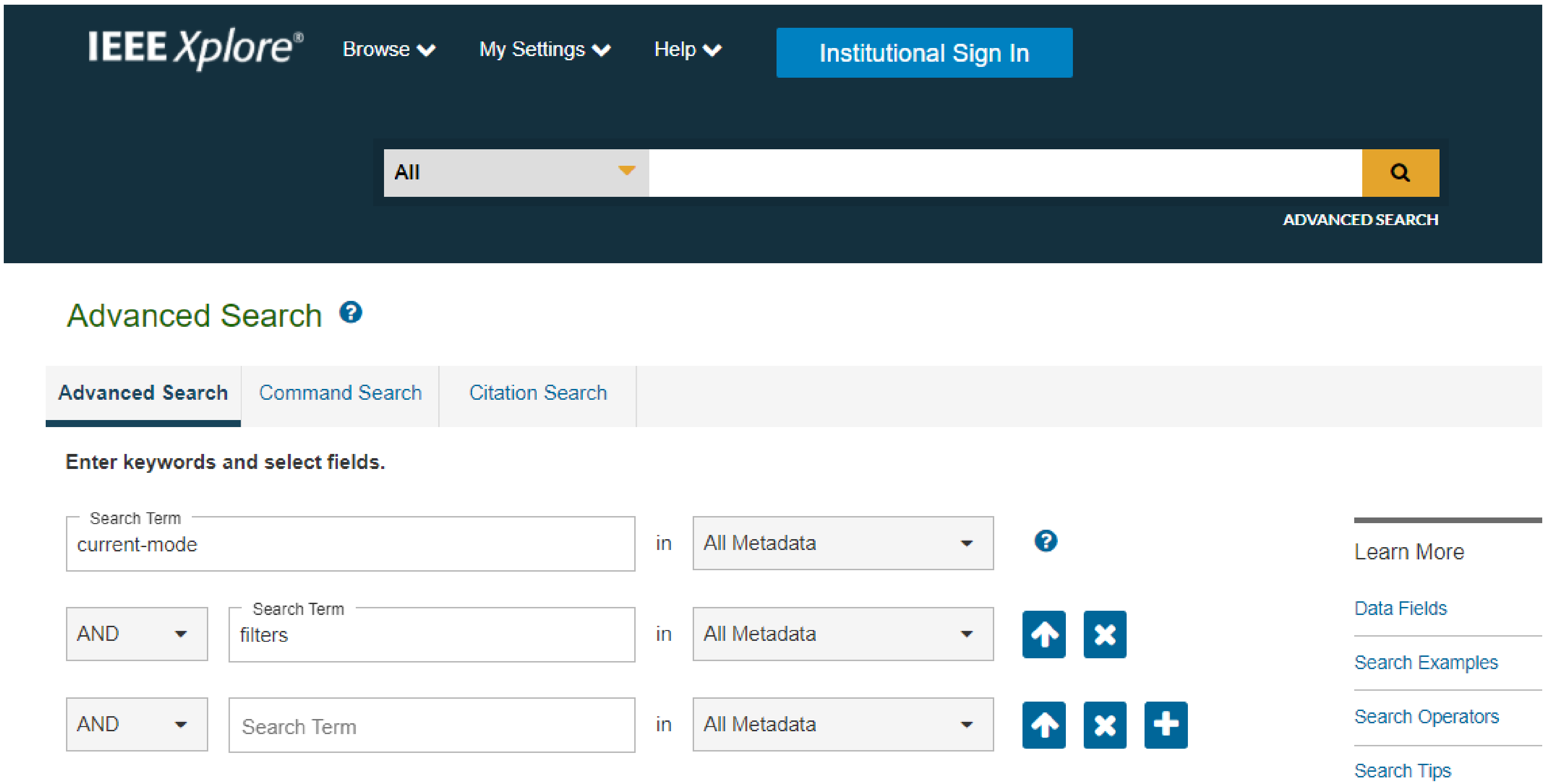
JB Hughes, NC Bird, IC Macbeth - … International Symposium on …, 1989 - ieeexplore.ieee.org

A technique called'**switched currents**,'for analog sampled-data signal processing in the current domain, is introduced. A family of modules that are capable of various computational and memory functions is described. The modules are well suited to system building as …

Oct 20 2020

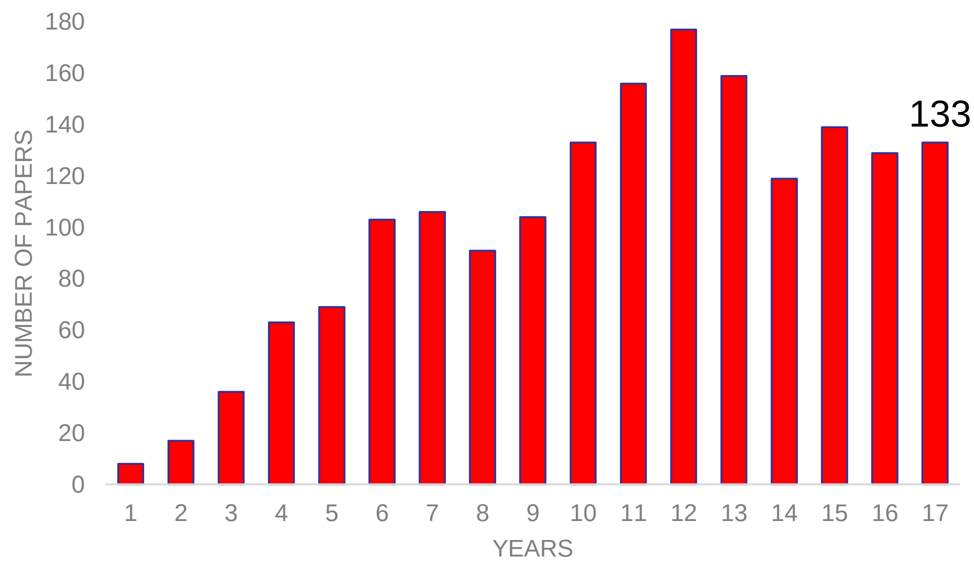
This paper is one of the most significant contributions that has ever come from ISCAS

Current-Mode Filters



Current-Mode Filters

Advanced Search for “current-mode” and “filters”



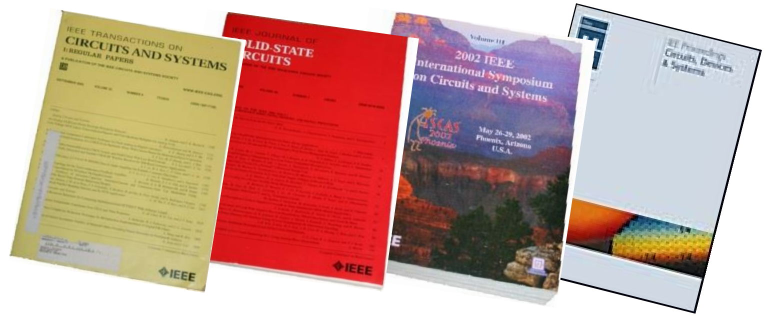
Histogram for 2-year intervals
Most recent is 2019-2019

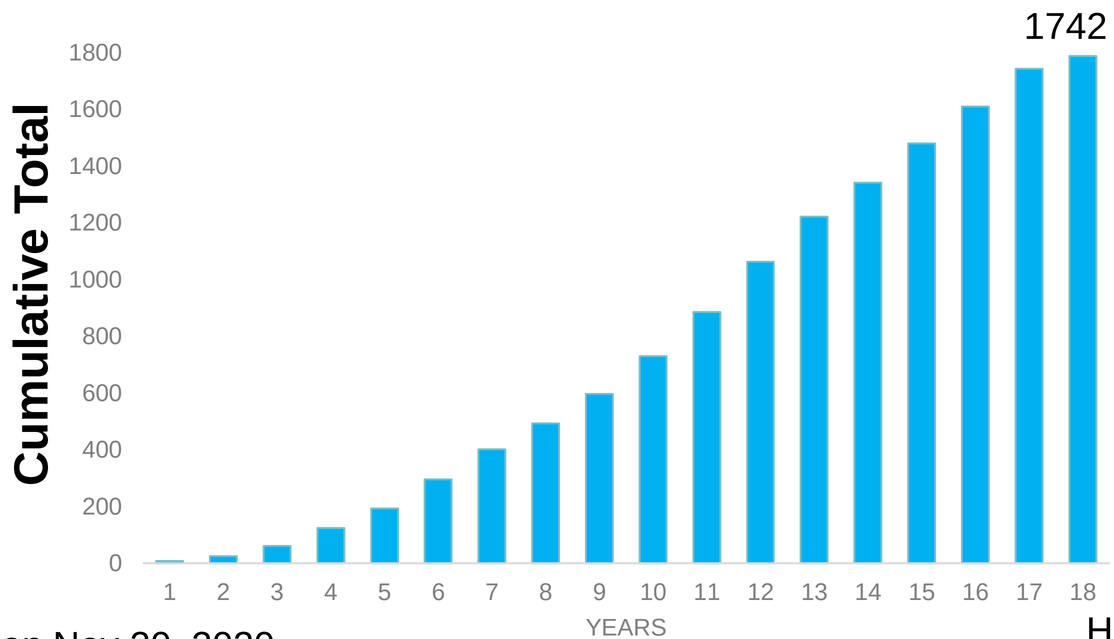
1872-1987 – total of 8 references

Search done on Nov 20, 2020

Current-Mode Filters

IEEE Xplore® RELEASE 2.4





Search done on Nov 20, 2020

Histogram for 2-year intervals
Most recent is 2019-2019

- Steady growth in research in the area since 1990 and publication rate is growing with time !!
- And growth is MUCH bigger outside of IEEE (e.g. Scholar)

Current-Mode Filters

The Conventional Wisdom:

Fully differential current-mode third-order Butterworth VHF G_m-C filter in 0.18 μlm CMOS

Download Citation | Email | Print

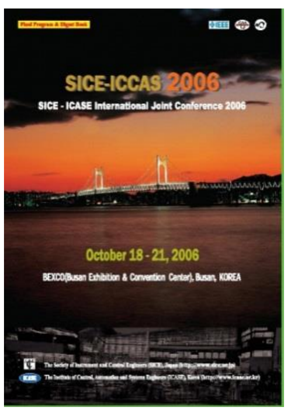
Hwang, Y.-S.; Chen, J.-J.; Lai, J.-H.; Sheu, P.-W.;
Dept. of Electron. Eng., Nat. Taipei Univ. of Technol., Taiwan

This paper appears in: Circuits, Devices and Systems, IEE Proceedings
Issue Date: Dec. 2006
Volume: 153 Issue:6
On page(s): 552 - 558

Proc. IEE Dec 2006:

1 Introduction

Current-mode circuits have been proven to offer advantages over their voltage-mode counterparts [1, 2]. They possess wider bandwidth, greater linearity and wider dynamic range. Since the dynamic range of the analogue circuits using low-voltage power supplies will be low, this problem can be solved by employing current-mode operation.



Proc. SICE-ICASE, Oct. 2006

1. INTRODUCTION

It is well known that current-mode circuits have been receiving significant attention owing to its advantage over the voltage-mode counterpart, particularly for higher frequency of operation and simpler filtering structure [1].

Current-controlled Current-mode Biquadratic Filter with two inputs and three outputs Using Multiple-Output FTFNs

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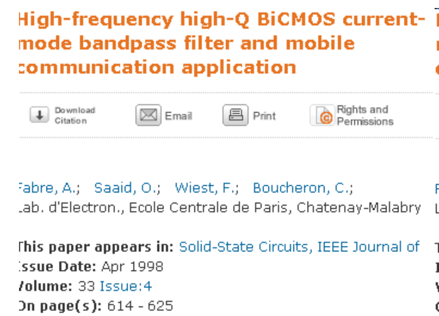
Hirunporm, J.; Pukkalanun, T.; Tangsrirat, W.;
Fac. of Eng., King Mongkut's Inst. of Technol., Bangkok

This paper appears in: SICE-ICASE, 2006. International Joint Conference
Issue Date: 18-21 Oct. 2006
On page(s): 5691 - 5694

Current-Mode Filters

The Conventional Wisdom:


High-frequency high-Q BiCMOS current-mode bandpass filter and mobile communication application

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abre, A.; Saaid, O.; Wiest, F.; Boucheron, C.;
ab. d'Electron., Ecole Centrale de Paris, Chatenay-Malabry

his paper appears in: Solid-State Circuits, IEEE Journal of
ssue Date: Apr 1998
olume: 33 Issue:4
n page(s): 614 - 625

JSC April 1998:

"… current-mode functions exhibit higher frequency potential, simpler architectures, and lower supply voltage capabilities than their voltage-mode counterparts."


Current-mode continuous-time filters: two design approaches

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Ramirez-Angulo, J. ; Robinson, M. ; Sanchez-Sinecio, E. ;
Dept. of Electr. & Comput. Eng., New Mexico State Univ., Las Cruces, NM

This paper appears in: Circuits and Systems II: Analog and Digital Signal Processing, IEEE Transactions on
Issue Date : Jun 1992
Volume : 39 , Issue:6
On page(s): 337 - 341

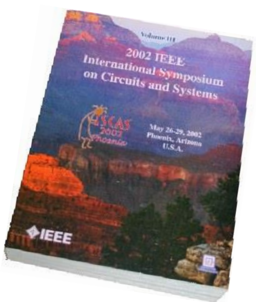
CAS June 1992

"Current-mode signal processing is a very attractive approach due to the simplicity in implementing operations such as … and the potential to operate at higher signal bandwidths than their voltage mode analogues" … "Some voltage-mode filter architectures using transconductance amplifiers and capacitors (TAC) have the drawback that …"

Current-Mode Filters

The Conventional Wisdom:

ISCAS 1993:



"In this paper we propose a fully balanced high frequency current-mode integrator for low voltage high frequency filters. Our use of the term current mode comes from the use of current amplifiers as the basic building block for signal processing circuits. This fully differential integrator offers significant improvement even over recently introduced circuit with respect to accuracy, high frequency response, linearity and power supply requirements. Furthermore, it is well suited to standard digital based CMOS processes."

3V high-frequency current-mode filters

Smith, S.L.; Sanchez-Sinencio, E.;

Circuits and Systems, 1993., ISCAS '93, 1993 IEEE International Symposium on

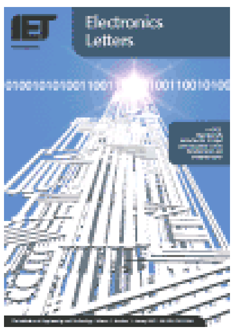
Digital Object Identifier: 10.1109/ISCAS.1993.394009

Publication Year: 1993 , Page(s): 1459 - 1462 vol.2

Current-Mode Filters

The Conventional Wisdom:

Two key publications where benefits of the current-mode circuits are often referenced:



All current-mode frequency selective circuits

GW Roberts, AS Sedra - Electronics Letters, 1989 - ieeexplore.ieee.org

Proposes an entirely new method for performing analogue signal filtering. **All** circuits are based upon current amplifiers but derived from well-known voltage amplifier circuits. These circuits possess the same sensitivity properties as their voltage amplifier counterparts ...

☆ 99 Cited by 493 Related articles All 8 versions

Search Nov 20, 2020

"To make greatest use of the available transistor bandwidth f_T, and operate at low voltage supply levels, it has become apparent that analogue signal processing can greatly benefit from processing current signals rather than voltage signals. Besides this, it is well known by electronic circuit designers that the mathematical operations of adding, subtracting or multiplying signals represented by currents are simpler to perform than when they are represented by voltages. This also means that the resulting circuits are simpler and require less silicon area."

Current-Mode Filters

The Conventional Wisdom:

Two key publications where benefits of the current-mode circuits are often referenced:

Recent developments in **current** conveyors and **current**-mode circuits

B Wilson - IEE Proceedings G (Circuits, Devices and Systems), 1990 - IET

… The **current conveyor**: history and progress', IEEE international symposium on circuits and systems, 1989, Portland, USA, 3, p. 1567–1570. 7): Toumazou, C., Lidgey, FJ, Cheung, PYK: `**Current**-mode analogue signal processing circuits — a review of **recent developments**', IEEE …

Cited by 735 Related articles All 4 versions

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“The **use** of current rather than voltage as the active parameter can result in higher usable gain, accuracy and bandwidth due to reduced voltage excursion at sensitive nodes. A current-mode approach is not just restricted to current processing, but also offers certain important advantages when interfaced to voltage-mode circuits.”

Current-Mode Filters

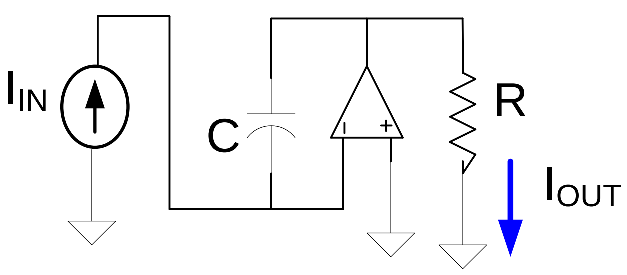
The Conventional Wisdom:

- Current-Mode circuits operate at higher-frequencies than voltage-mode counterparts
- Current-Mode circuits operate at lower supply voltages and lower power levels than voltage-mode counterparts
- Current-Mode circuits are simpler than voltage-mode counterparts
- Current-Mode circuits offer better linearity than voltage-mode counterparts

This represents four really significant benefits of current-mode circuits!

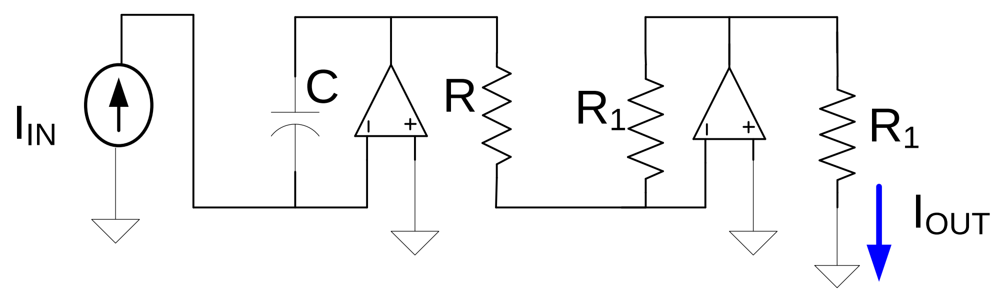
Some Current-Mode Integrators

Active RC



$$I_{OUT} = \left(\frac{-1}{RCs}\right) I_{IN}$$

Inverting



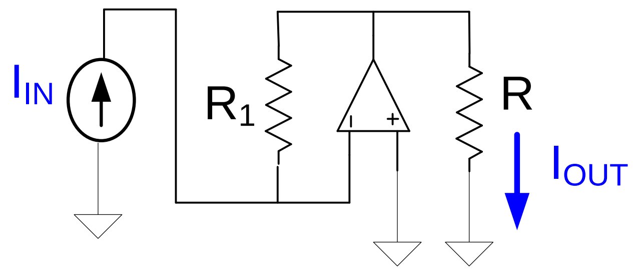
$$I_{OUT} = \left(\frac{1}{RCs}\right) I_{IN}$$

Noninverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Same component count as voltage-mode integrators
- Some argue that since only interested in currents, can operate at lower voltages

Some Current-Mode Integrators

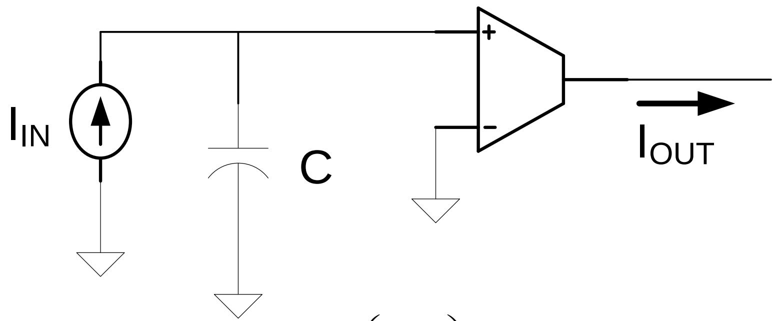
Current-Mode Inverting Amplifier



$$I_{OUT} = \left(-\frac{R_1}{R} \right) I_{IN}$$

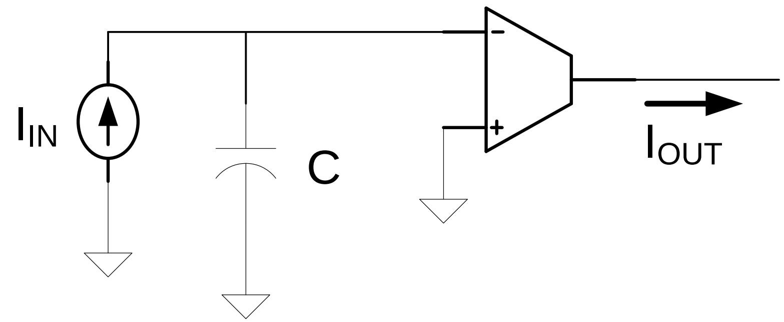
Some Current-Mode Integrators

OTA-C



$$I_{OUT} = \left(\frac{g_m}{Cs}\right) I_{IN}$$

Noninverting



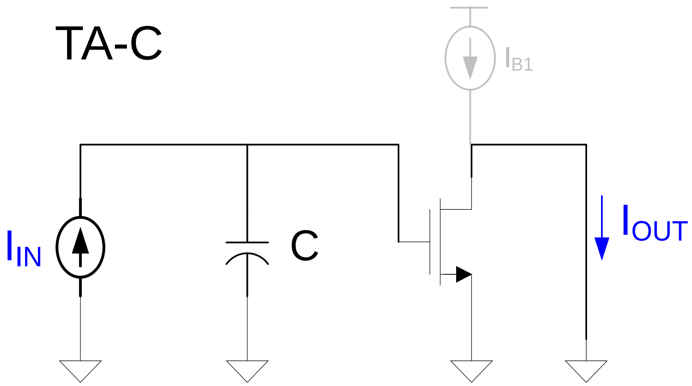
$$I_{OUT} = \left(\frac{-g_m}{Cs}\right) I_{IN}$$

Inverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Same component count as voltage-mode integrators
- Many argue that since only interested in currents, can operate at lower voltages and higher frequencies

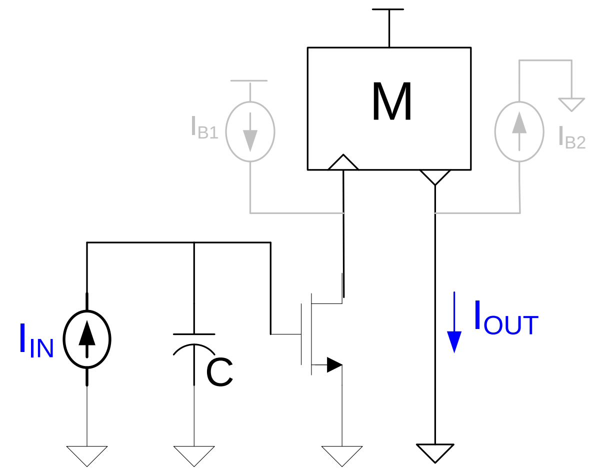
Some Current-Mode Integrators

TA-C



$$I_{OUT} = \left(\frac{-g_m}{Cs}\right) I_{IN}$$

Inverting



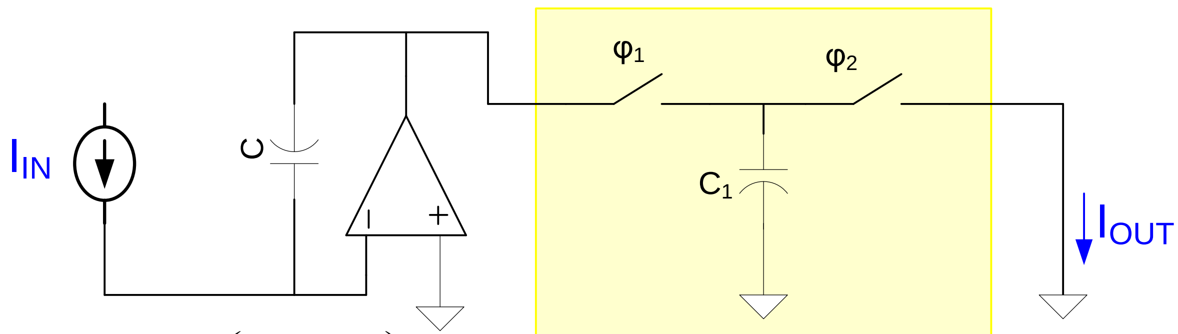
$$I_{OUT} = \left(\frac{g_m}{Cs}\right) I_{IN}$$

Noninverting

- Summing inputs really easy to obtain
- Loss is easy to add
- Same component count as voltage-mode integrators
- Many argue that since only interested in currents, can operate at lower voltages and higher frequencies

Some Current-Mode Integrators

Switched-C

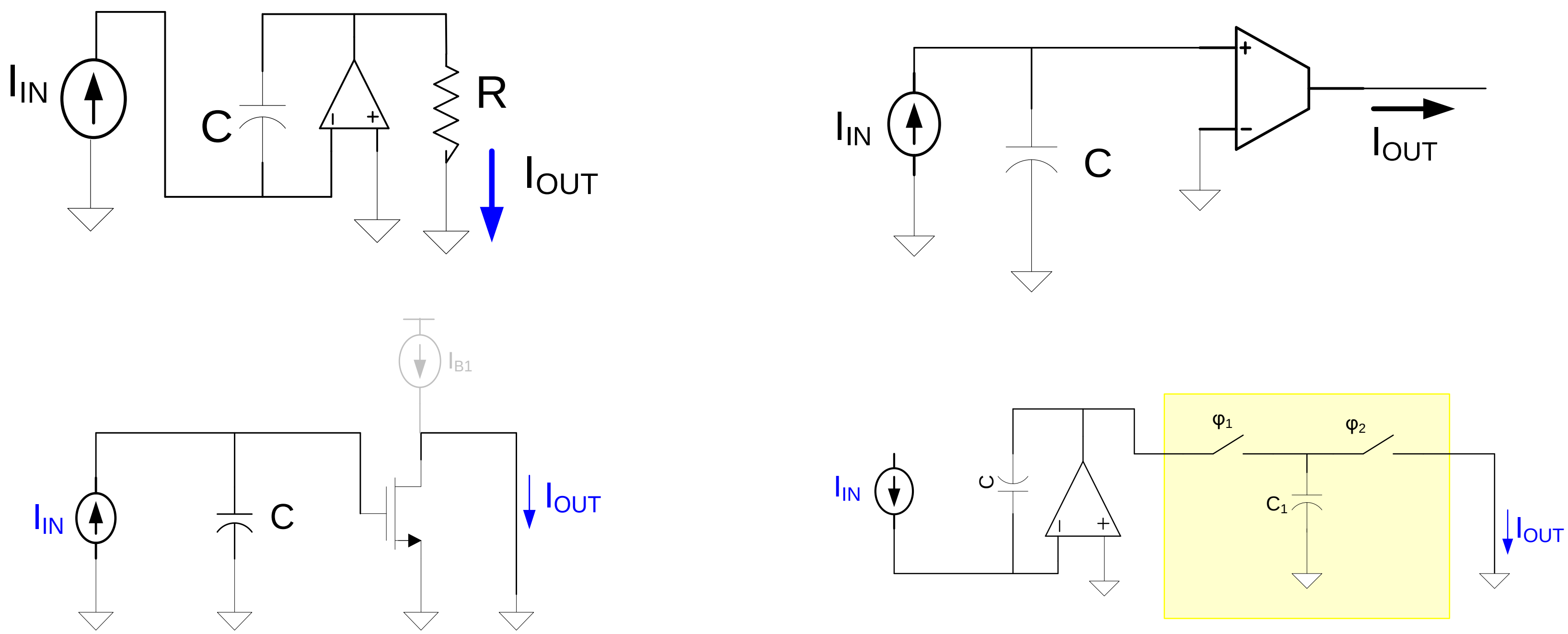


$$I_{OUT} = \left(\frac{-1}{R_{EQ} C s} \right) I_{IN}$$

Inverting

- Noninverting input easy to obtain
- Summing inputs really easy to obtain
- Loss is easy to add
- Stray insensitive structures readily available
- Less component count than voltage-mode integrators because summing input requires no additional inputs
- SC current-mode integrators have not received much attention in the literature (likely because few have observed the equivalence noted above)

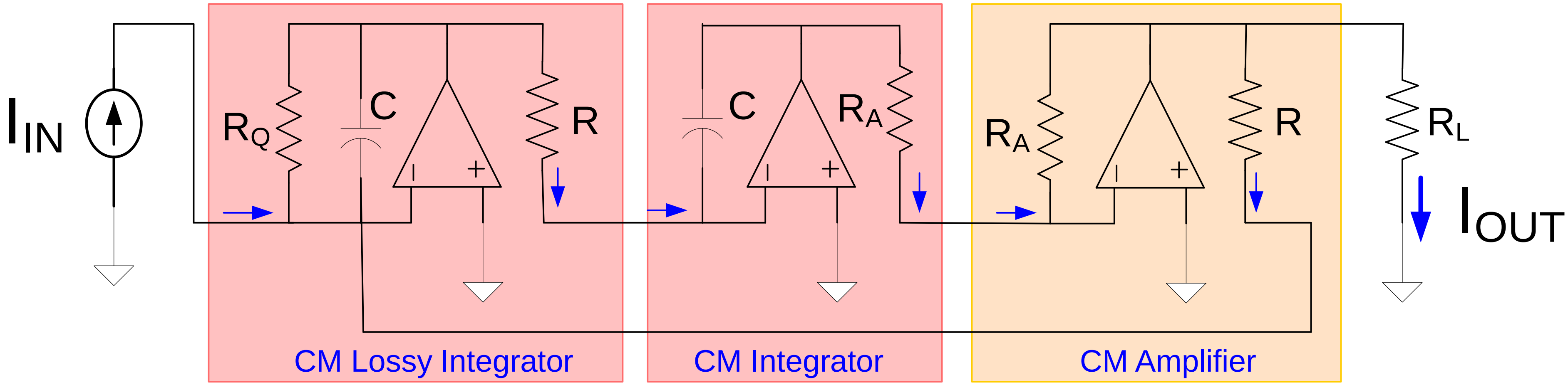
Current-Mode Integrators



The other basic types of voltage-mode integrators also have current-mode counterparts

- Switched-resistor
- MOSFET-C
- “Other”

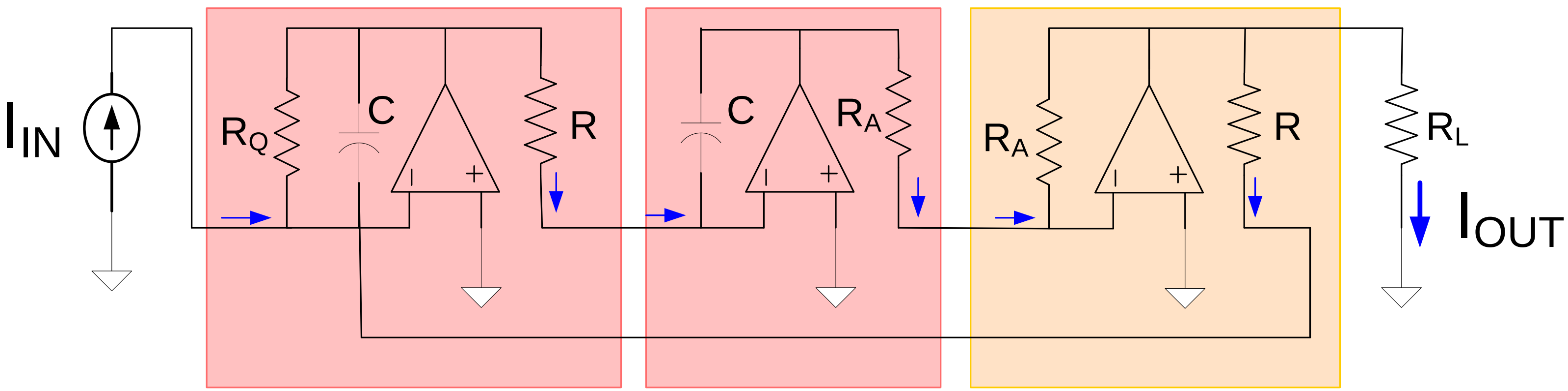
Current-Mode Two Integrator Loop

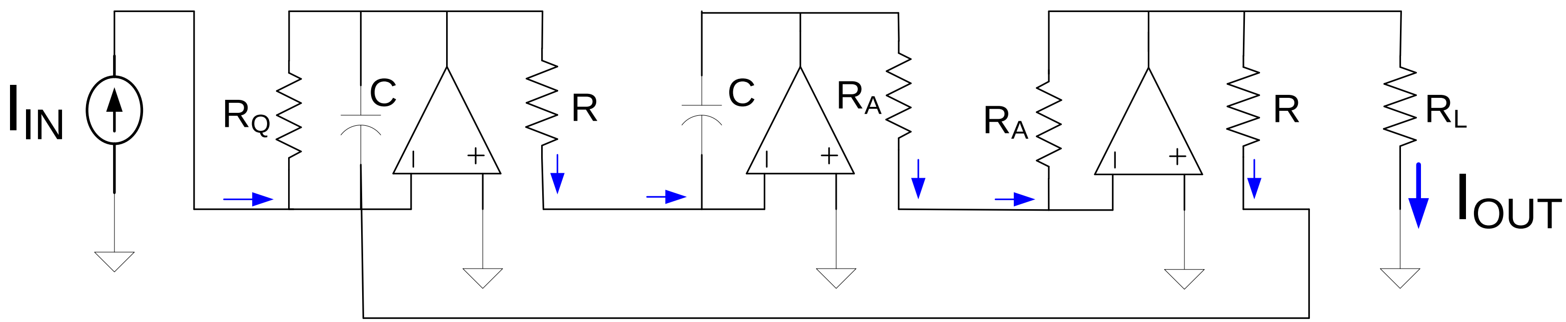


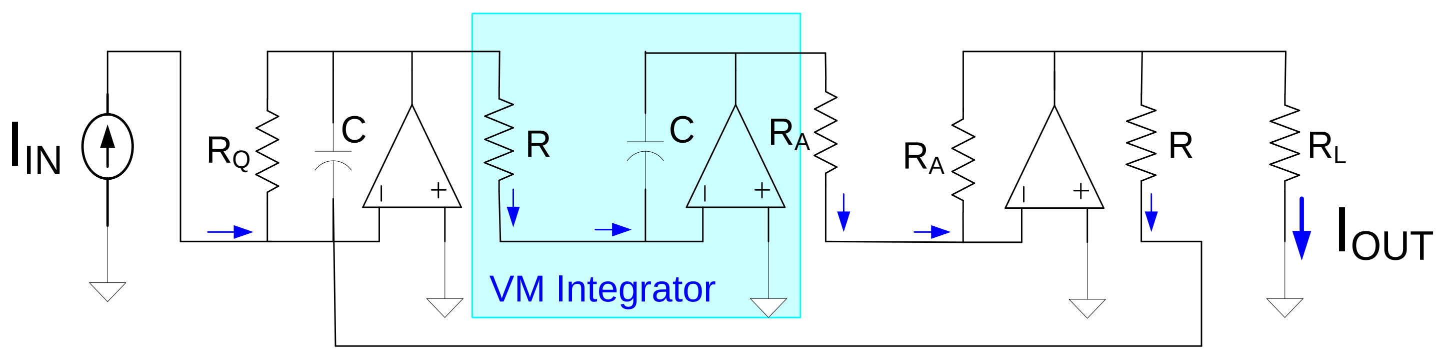
- Straightforward implementation of the two-integrator loop
- Simple structure

Current-Mode Two Integrator Loop

An Observation:

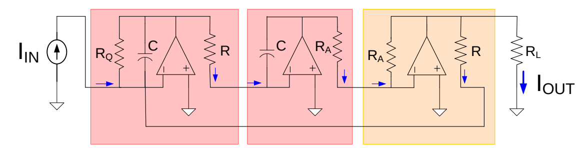


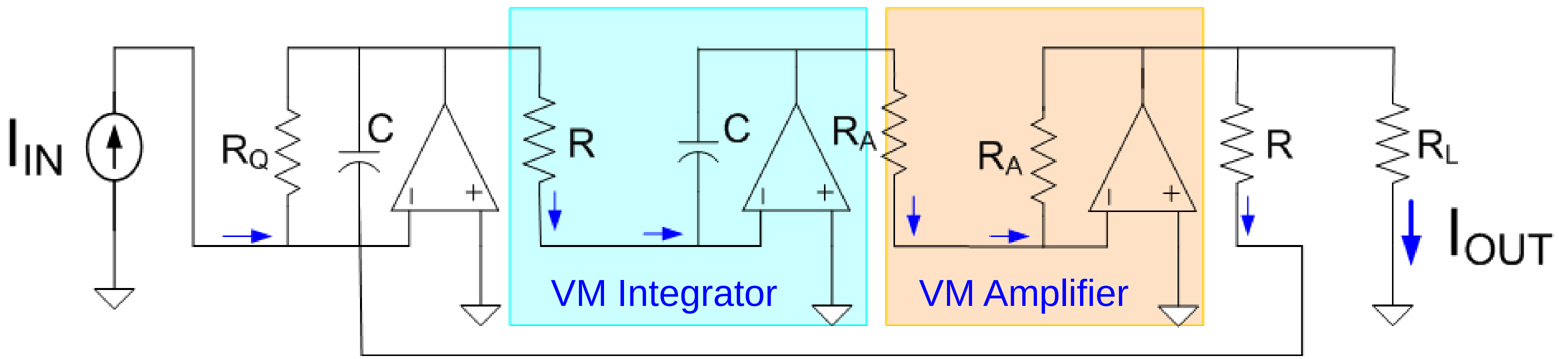


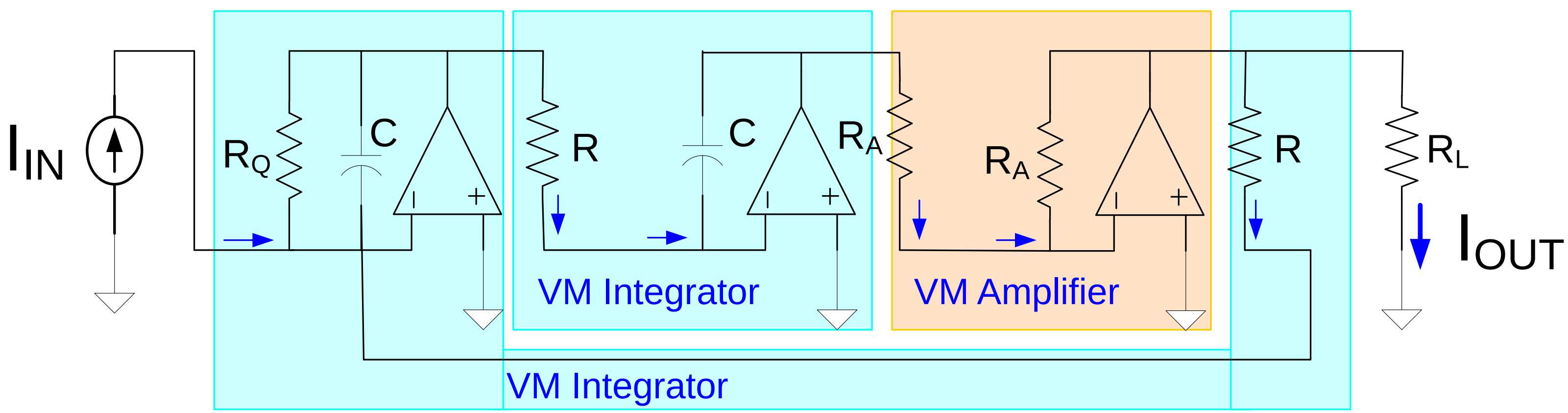


Current-Mode Two Integrator Loop

An Observation:







This circuit is identical to another one with two voltage-mode integrators and a voltage-mode amplifier !

Analog Integrated Circuits and Signal Processing, 38, 83–101, 2004

Current Mode, Voltage Mode, or Free Mode? A Few Sage Suggestions

BARRIE GILBERT
Analog Devices Inc., 1100 NW Compton Drive, Beaverton, Oregon, USA
E-mail: barrie.gilbert@analog.com

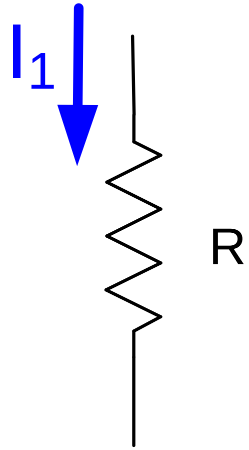
Yet, to the author's knowledge, no *rigorous* definition of a current-mode circuit can be found in the literature. Since the term has been so widely employed, this should strike us as rather surprising.

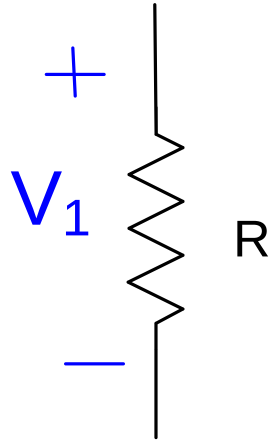
This ambiguity can be attributed to a casual, and often inappropriate, appeal to the term in publications,at times with insufficient regard for foundations developed decades ago, referencing instead recent and closely similar work. Its colloquial application implies the use of *currents as signals*, invariably with a tacit claim to a degree of *novelty*, announcing a *different*, and in some (unstated and unclear) way, *advantageous* implementation of a function formerly realized using other techniques.

What is a current-mode circuit or a current-mode device?

Some authors say a current-mode circuit processes signals in the current domain

Is the resistor a current-mode device?





Observation

- Many papers have appeared that tout the performance advantages of current-mode circuits
- In all of the current-mode papers that this instructor has seen, no attempt is made to provide a quantitative comparison of the key performance features of current-mode circuits with voltage-mode counterparts
- All justifications of the advantages of the current-mode circuits this instructor has seen are based upon qualitative statements

Observations (cont.)

- It appears easy to get papers published that have the term “current-mode” in the title
- Over 1700 papers have been published in IEEE forums alone !
- Some of the “current-mode” filters published perform better than other “voltage-mode” filters that have been published
- We are still waiting for even one author to quantitatively show that current-mode filters offer even one of the claimed four advantages over their voltage-mode counterparts

Will return to a discussion of Current-Mode filters later

Switched-Current Filters

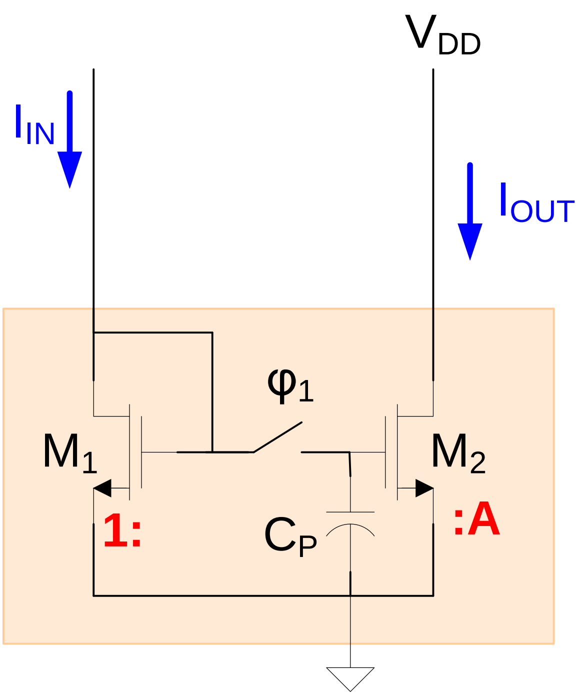
Basic idea introduced by Hughes and Bird at ISCAS 1989

Switched currents-a new technique for analog sampled-data signal processing
JB Hughes, NC Bird... - Circuits and Systems, 1989 ..., 2002 - ieeexplore.ieee.org
NTRODUCTION The enormous complexity available in state-of-the-art CMOS processing has made possible the integration of complete systems, including both digital and analog signal processing functions, within the same chip Through the last decade, the **switched** capacitor technique **...**
Cited by 151 - Related articles

Technique introduced directly in the z-domain

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989



$$I_{OUT} = \begin{cases} AI_{IN}(t) & \text{for } \varphi_1 \text{ closed} \\ AI_{IN}(T_{SW}) & \text{for } \varphi_1 \text{ open} \end{cases}$$

If Φ_1 is a periodic signal and if I_{IN} is also appropriately clocked, the input/output currents of this circuit can be represented with the difference equation

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$

This switched mirror becomes a delay element

"Gain" A is that of a current mirror

A can be accurately controlled

Circuit is small and very fast

Concept can be extended to implement arbitrary difference equation

Difference equation characterizes filter H(z)

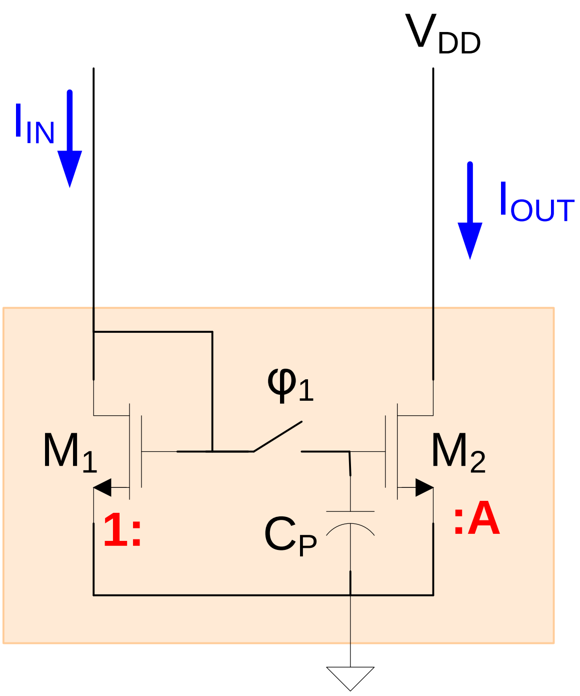
Need only current mirrors and switches

Truly a "current-mode" circuit

Switched-Current Filters

Basic idea introduced by Hughes and Bird at ISCAS 1989

$$I_{OUT}(nT) = AI_{IN}(nT-T)$$



Cp is parasitic gate capacitance on M_2

Very low power dissipation

Potential to operate at very low voltages

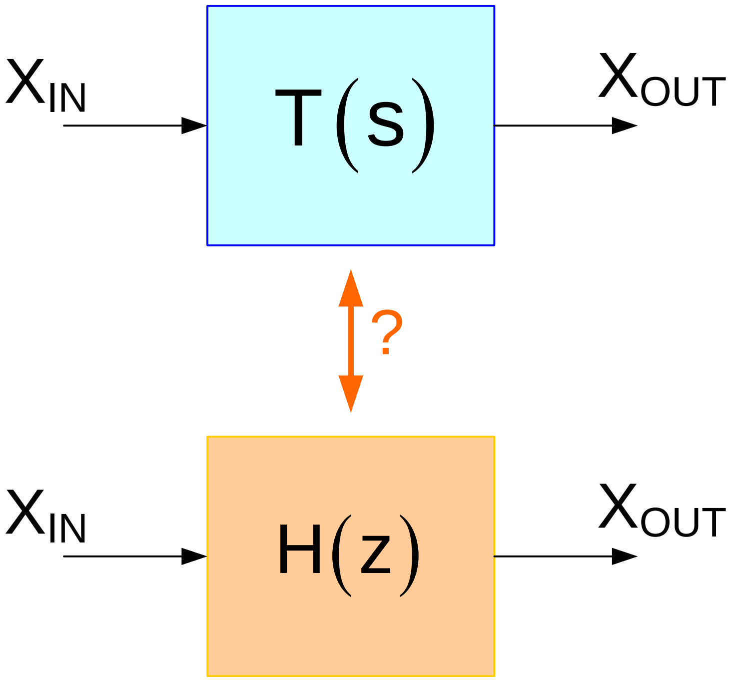
Potential for accuracy of a SC circuit at both low and high frequencies but without the Op Amp and large C ratios

Neither capacitor or resistor values needed to do filtering!

A completely new approach to designing filters that offers potential for overcoming most of the problems plaguing filter designers for decades !

Before developing Switch-Current concept, need to review background information in s to z domain transformations

s-domain to z-domain transformations

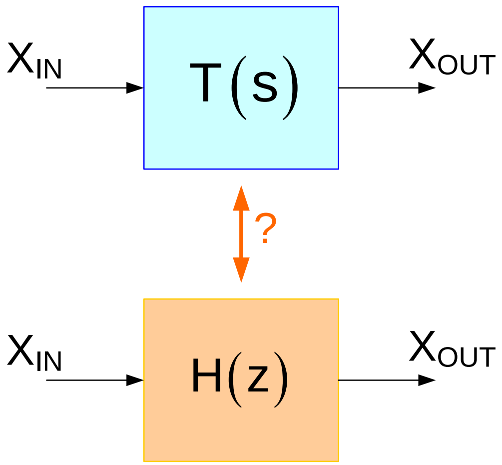


For a given T(s) would like to obtain a function H(z) or for a given H(z) would like to obtain a T(s) that preserves the magnitude and phase response

Mathematically, would like to obtain the relationship:

$$T(s)\Big|_{s=j\omega} = H(z)\Big|_{z=e^{j\omega T}}$$

s-domain to z-domain transformations



want: $$T(s)\big|_{s=j\omega} = H(z)\big|_{z=e^{j\omega T}}$$

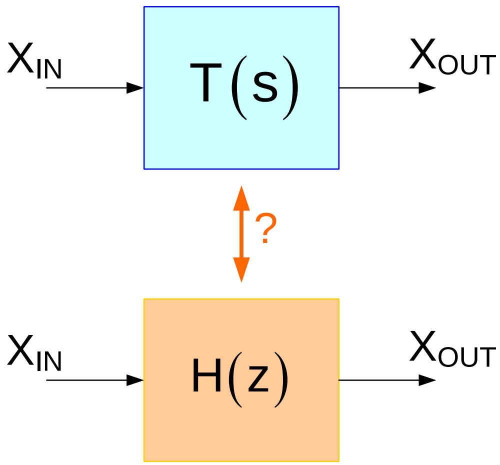
equivalently, want:

$$T(s) = H(z)\big|_{z=e^{sT}}$$

But if this were to happen, T(s) would not be a rational fraction in s with real coeff.

Thus, it is impossible to obtain this mapping between T(s) and H(z)

s-domain to z-domain transformations



goal: $$T(s) = H(z)\big|_{z=e^{sT}}$$

If can't achieve this goal, would like to map imaginary axis to unit circle and map stable filters to stable filters

consider: $$z = e^{sT}$$

Case 1:

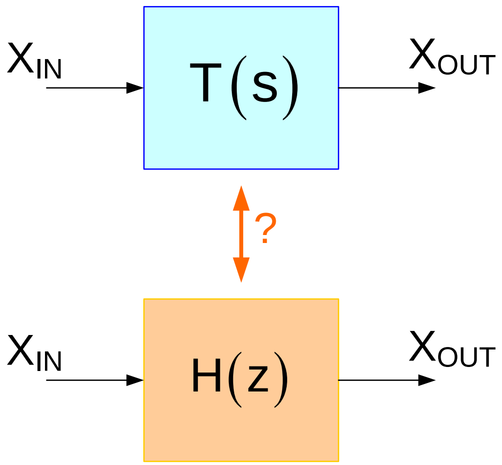
$$z = e^{sT} \cong \sum_{i=0}^{\infty} \frac{1}{i!}(sT)^i$$

$$z = \sum_{i=0}^{\infty} \frac{1}{i!}(sT)^i \cong 1 + sT$$

$$s = \frac{z - 1}{T}$$

Termed the Forward Euler transformation

s-domain to z-domain transformations

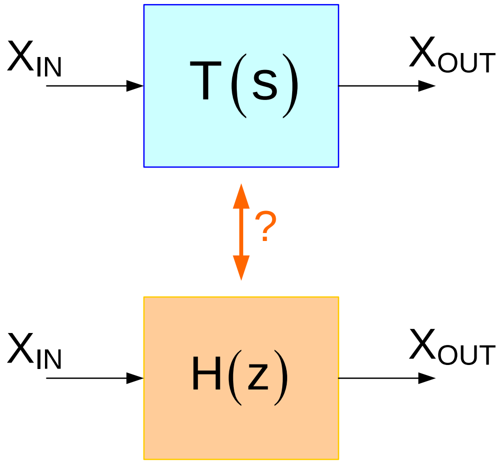


$$s = \frac{z - 1}{T}$$

Forward Euler transformation

- Doesn't map imaginary axis in s-plane to unit circle in z-plane
- Doesn't <u>guarantee</u> stable filter will map to stable filter
- But mapping may give stable filter with good frequency response

s-domain to z-domain transformations



consider: $z = e^{sT}$

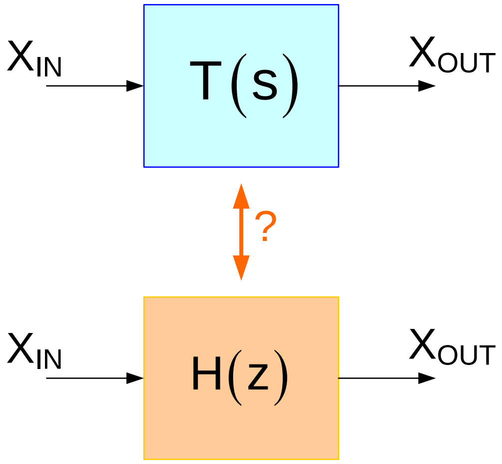
Case 2: $z = e^{sT} = \dfrac{1}{e^{-sT}} = \dfrac{1}{\sum_{i=0}^{\infty} \frac{1}{i!}(-sT)^i} \cong \dfrac{1}{1-sT}$

$$z \cong \frac{1}{1-sT}$$

$$s = \left(\frac{1}{T}\right)\frac{z-1}{z}$$

Termed the Backward Euler transformation

s-domain to z-domain transformations

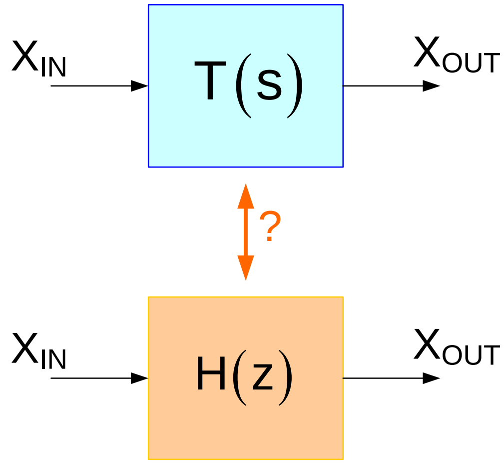


$$s = \left(\frac{1}{T}\right)\frac{z-1}{z}$$

Backward Euler transformation

- Doesn't map imaginary axis in s-plane to unit circle in z-plane
- Does <u>guarantee</u> stable filter will map to stable filter

s-domain to z-domain transformations



consider: $z = e^{sT}$

Case 3:

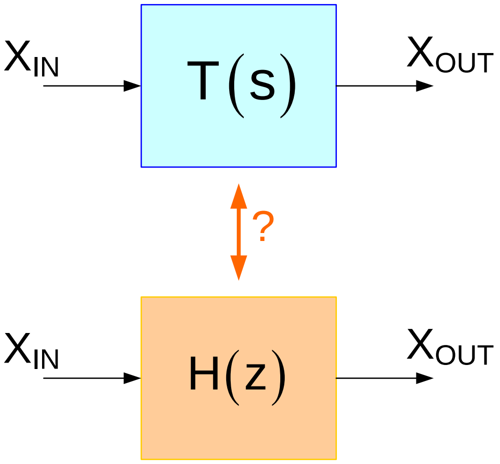
$$z = e^{sT} = \frac{e^{s\frac{T}{2}}1}{e^{-s\frac{T}{2}}} = \frac{\sum_{i=0}^{\infty}\frac{1}{i!}\left(s\frac{T}{2}\right)^{i}}{\sum_{i=0}^{\infty}\frac{1}{i!}\left(-s\frac{T}{2}\right)^{i}} \cong \frac{1+s\frac{T}{2}}{1-s\frac{T}{2}}$$

solving for s, obtain

$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$

Termed the Bilinear z transformation

s-domain to z-domain transformations

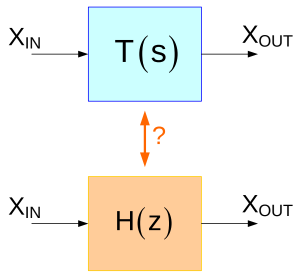


$$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$$

Bilinear z transformation

- Maps imaginary axis in s-plane to unit circle in z-plane (preserves shape, distorts frequency axis)
- Does <u>guarantee</u> stable filter will map to stable filter
- Bilinear z transformation is widely used

s-domain to z-domain transformations

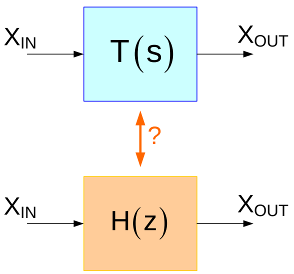


consider: $z = e^{sT}$

Three Popular Transformations

$s = \dfrac{z-1}{T}$	Forward Euler	$s = \dfrac{1 - z^{-1}}{Tz^{-1}}$
$s = \dfrac{z-1}{Tz}$	Backward Euler	$s = \dfrac{1-z^{-1}}{T}$
$s = \dfrac{2}{T} \bullet \dfrac{z-1}{z+1}$	Bilinear z transform	$s = \dfrac{2}{T} \bullet \dfrac{1-z^{-1}}{1+z^{-1}}$

s-domain to z-domain transformations

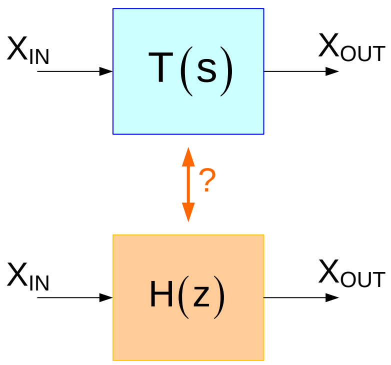


Three Popular Transformations

$s = \frac{z-1}{T}$	Forward Euler	$s = \frac{1-z^{-1}}{Tz^{-1}}$
$s = \frac{z-1}{Tz}$	Backward Euler	$s = \frac{1-z^{-1}}{T}$
$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$	Bilinear z transform	$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$

- Transformations of standard approximations in s-domain are the corresponding transformations in the z-domain
- Transformations are not unique
- Transformations cause warping of the imaginary axis and may cause change in basic shape
- Transformations do not necessarily guarantee stability
- These transformations preserve order

z-domain integrators



$$T(s) = \frac{I_0}{s}$$

Some z-domain integrators

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1} & \text{Forward Euler} \\ \dfrac{I_0 T z}{z-1} & \text{Backward Euler} \\ \dfrac{TI_0}{2}\left(\dfrac{z+1}{z-1}\right) & \text{Bilinear z} \end{cases}$$

Three Popular Transformations

$s = \dfrac{z-1}{T}$	Forward Euler	$s = \dfrac{1-z^{-1}}{Tz^{-1}}$
$s = \dfrac{z-1}{Tz}$	Backward Euler	$s = \dfrac{1-z^{-1}}{T}$
$s = \dfrac{2}{T} \bullet \dfrac{z-1}{z+1}$	Bilinear z transform	$s = \dfrac{2}{T} \bullet \dfrac{1-z^{-1}}{1+z^{-1}}$

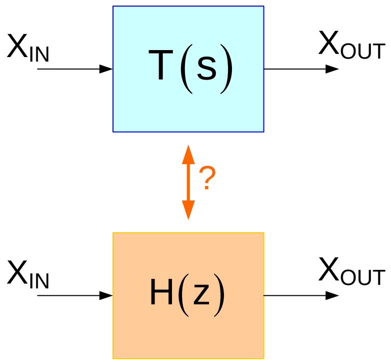
Corresponding difference equations:

$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + V_{OUT}(nT) \qquad \text{Forward Euler}$$

$$V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT) \qquad \text{Backward Euler}$$

$$V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + V_{OUT}(nT) \qquad \text{Bilinear z}$$

z-domain lossy integrators



$$T(s) = \frac{I_0}{s+\alpha}$$

Three Popular Transformations

$s = \frac{z-1}{T}$	Forward Euler	$s = \frac{1-z^{-1}}{Tz^{-1}}$
$s = \frac{z-1}{Tz}$	Backward Euler	$s = \frac{1-z^{-1}}{T}$
$s = \frac{2}{T} \bullet \frac{z-1}{z+1}$	Bilinear z transform	$s = \frac{2}{T} \bullet \frac{1-z^{-1}}{1+z^{-1}}$

Some z-domain lossy integrators

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1+\alpha T} & \text{Forward Euler} \\ \dfrac{I_0 Tz}{z(1+\alpha T)-1} & \text{Backward Euler} \\ \dfrac{TI_0}{2}\left(\dfrac{z+1}{z\left(1+\dfrac{\alpha T}{2}\right)+\left(\dfrac{\alpha T}{2}-1\right)}\right) & \text{Bilinear z} \end{cases}$$

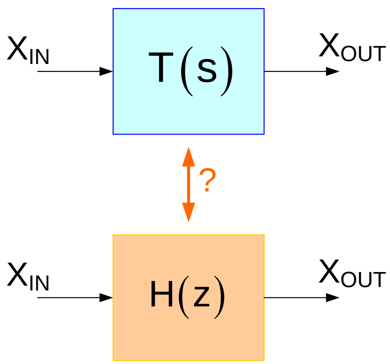
Corresponding difference equations:

$$V_{OUT}(nT+T) = TI_0 V_{IN}(nT) + [1-\alpha T] V_{OUT}(nT) \qquad \text{Forward Euler}$$

$$(1+\alpha T) V_{OUT}(nT+T) = I_0 T V_{IN}(nT+T) + V_{OUT}(nT) \qquad \text{Backward Euler}$$

$$\left(1+\frac{\alpha T}{2}\right) V_{OUT}(nT+T) = \frac{TI_0}{2}\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + \left[1-\frac{\alpha T}{2}\right] V_{OUT}(nT) \qquad \text{Bilinear z}$$

z-domain lossy integrators



Some z-domain lossy integrators

$$T(s) = \frac{I_0}{s+\alpha}$$

Functional Form

$$H(z) = \begin{cases} \dfrac{TI_0}{z-1+\alpha T} & \dfrac{G}{z-H} \quad \text{Forward Euler} \\[2ex] \dfrac{I_0 Tz}{z(1+\alpha T)-1} & \dfrac{Gz}{zH-1} \quad \text{Backward Euler} \\[2ex] \dfrac{TI_0}{2}\left(\dfrac{z+1}{z\left(1+\dfrac{\alpha T}{2}\right)+\left(\dfrac{\alpha T}{2}-1\right)}\right) & G\left(\dfrac{z+1}{z-H}\right) \quad \text{Bilinear z} \end{cases}$$

Corresponding difference equations:

$$V_{OUT}(nT+T) = GV_{IN}(nT) + HV_{OUT}(nT) \qquad \text{Forward Euler}$$

$$HV_{OUT}(nT+T) = GV_{IN}(nT+T) + V_{OUT}(nT) \qquad \text{Backward Euler}$$

$$V_{OUT}(nT+T) = G\left(V_{IN}(nT+T) + V_{IN}(nT)\right) + HV_{OUT}(nT) \qquad \text{Bilinear z}$$

Stay Safe and Stay Healthy !

End of Lecture 28